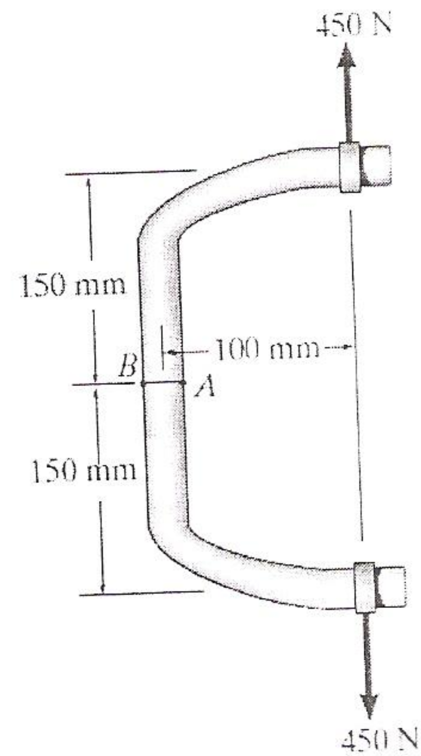
MECH 320-MECHANICS OF MATERIALS-FINAL EXAM (2H)

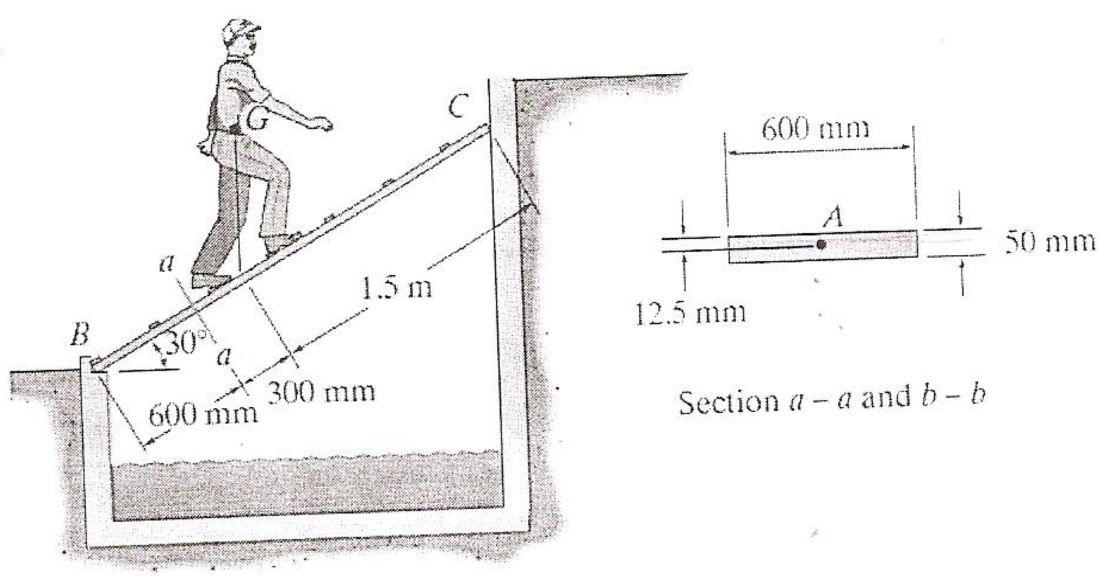
Problem 1. Determine the principal stresses and the maximum in-plane shear stress that are developed at point *A*. Show the results on an element located at this point. The rod has a diameter of 40 mm. (25 pts)

Steps:

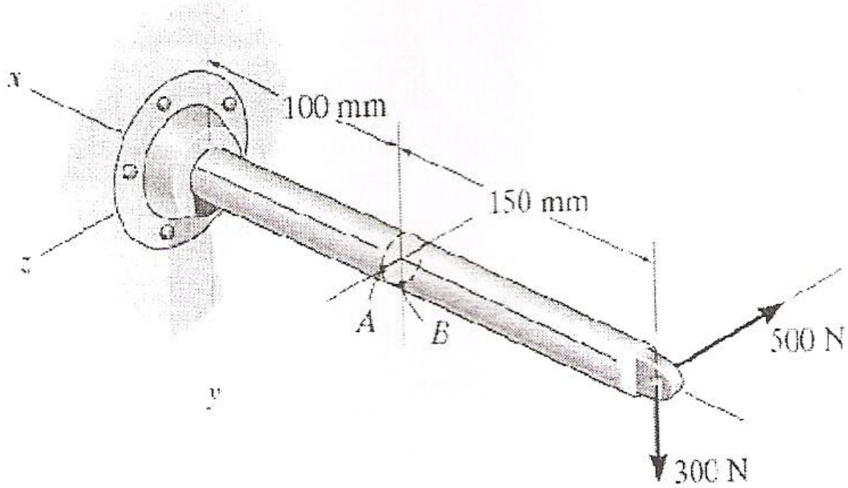
1. Using the method of sections and consider the FBD of the member's upper cut segment, find the normal stress which is the combination of axial and bending stress.
2. At point A, find σ and τ and draw the state of stress.
3. After setting the values of σ_x, σ_y *and* τ_{xy} find σ_{ave}
4. Draw the Mohr circle and specify σ_1, σ_2, and $\tau_{in-plane}^{max}$
5. Draw the state of maximum In - Plane shear stress on an element rotated through θ_s



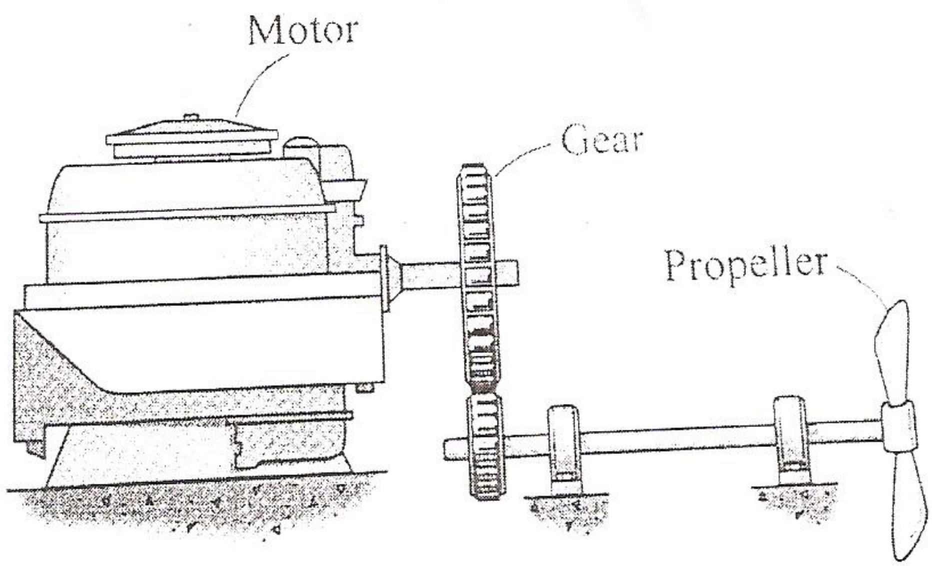
Problem 2. If the 75-kg man stands in the position shown, determine the state of stress at point *A* on the cross section of the plank at section *a–a*. The center of gravity of the man is at *G*. Assume that the contact point at *C* is smooth. (25 pts)



Problem 3. The bar has a diameter of 40 mm. If it is subjected to the two force components at its end as shown, determine the stress components that act at points *A and B,* and show the results on 2 volume elements located at these 2 points. (25 pts)



Problem 4. A diesel engine for a small commercial boat operates at 200 rpm and delivers 800 hp (hp=Horse power, 1hp=33,000 lb.ft/min) through a gear box with a ratio of 4 to 1 to the propeller as shown in the figure. Both the shaft from the engine to the gearbox and the propeller shaft are to be solid and made of heat-treated alloy steel. Determine the minimum permissible diameters for the two shafts if the allowable shearing stress is 20 ksi and the angle of twist in a 10-ft length of the propeller is not to exceed 4°. Neglect power loss in the gearbox and assume that the propeller shaft is subjected to pure torsion. (25 pts)



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Bonus (10 pts). Follow The four integrations procedure to calculate the deflections of a beam

Formulas:

- Torsional Stresses: $\tau = \left(\frac{\rho}{c}\right)\tau_{max}$ $\tau_{max} = \frac{T\,c}{J}$ $\tau = \frac{T\,\rho}{J}$ $J = \int_A \rho^2\, dA = \frac{\pi}{2}c^4$
- Torsional Angle of Twist: $\phi = \frac{T\,L}{J\,G}$
- Torsion - Gear Compatibility: $\phi_1\,\rho_1 = \phi_1\,\rho_2$
- Pure Bending - Normal Strain: $\varepsilon_x = -\frac{y}{\rho}$ $\varepsilon_{max} = c/\rho$ $\varepsilon_x = -\frac{y}{c}\varepsilon_m$
- Pure Bending - Normal Stress: $\sigma_x = -\frac{y}{c}\sigma_m$ $\sigma_x(y) = -\frac{M\,y}{I}$ $\sigma_{max} = \frac{M\,c}{I}$
- Section Properties: $I = \int_A y^2\, dA$; $I = \sum_i \left(I_i + A_i\, d_i^2\right)$; Centroid: $\int_A y\, dA = 0$; $\bar{y}\,A = \sum_i y_i\, A_i$
- Biaxial Bending: $\sigma_x = -\frac{M_z\, y}{I_z} + \frac{M_y\, z}{I_y}$; $\tan\phi = \frac{I_z}{I_y}\tan\theta$; $M_z = M\cos\theta$; $M_y = M\sin\theta$
- Eccentric Axial Loading: $\sigma_x = \frac{P}{A} - \frac{M\,y}{I}$;
- Shear Flow: $q = V\,Q/I$
- Flexural Shear Stress: $\tau_{ave} = \frac{V\,Q}{I\,t}$; $Q = \int_A y\, dA = A\,\bar{y}$
- Discrete Fasteners: $F_N = q \times s$
- Plane Stress Transformations: $\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos(2\theta) + \tau_{xy}\sin(2\theta)$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos(2\theta) - \tau_{xy}\sin(2\theta)\ ;\quad \tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin(2\theta) + \tau_{xy}\cos(2\theta)$$

- Principal Normal Stress: $\sigma_{p1,p2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$: $\tan(2\theta_p) = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2}$
- Maximum Shear Stress: $\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$; $\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}$; $\tan(2\theta_s) = -\frac{(\sigma_x - \sigma_y)/2}{\tau_{xy}}$
- Mohr's Circle: $\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}$; $R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$; $\sigma_{p1,p2} = \sigma_{avg} \pm R$; $\tau_{max} = R$

$J = \frac{\pi}{2}c^4$ **(Solid cross-section)**

$I = \frac{1}{12}bh^3\ (Rectangular)$